Int. J. Multiphase Flow, Vol. 2, pp. 591–595. Pergamon/Elsevier, 1976. Printed in Great Britain.

BRIEF COMMUNICATION

A THEORETICAL APPROACH TO THE LOCKHART-MARTINELLI CORRELATION FOR STRATIFIED FLOW

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(Received 16 July 1975)

1. INTRODUCTION

The work of Lockhart & Martinelli (1949) is one of the earliest attempts to model the functional pressure drop for two phase gas liquid flow. Their basic contribution lies in the suggestion that the dimensionless pressure drop ϕ_G or ϕ_L and the hold up R are unique functions of the parameter X where $X^2 = (\mathrm{d}P/\mathrm{d}x)_{Ls}/(\mathrm{d}P/\mathrm{d}x)_{Gs}$ is the ratio of the frictional pressure gradient of the liquid to that of the gas when each phase flows along in the pipe. The subscripts L and G refer to the liquid and gas respectively, the subscript s designates "superficial" or the situation where the designated phase flows alone in the pipe. Likewise ϕ_G and ϕ_L are defined by $\phi_G{}^2 = (\mathrm{d}P/\mathrm{d}x)/(\mathrm{d}P/\mathrm{d}x)_{Gs}$ and $\phi_L{}^2 = (\mathrm{d}P/\mathrm{d}x)/(\mathrm{d}P/\mathrm{d}x)_{Ls}$. $\mathrm{d}P/\mathrm{d}x$ is the frictional pressure gradient when both phases flow together. The resulting correlation rests strongly on a large set of experimental data and only weakly on physical modelling. In reality the existence of the relationship between ϕ and X is a confirmation of an ingeneous assumption. However, the correlation which results contains considerable scatter and shows systematic trends with flow rates and other operating conditions if applied over a wide range of flow conditions.

It is interesting that the assumption that ϕ_G is a unique function of X can be proven analytically for the case of separated flow. However, it was not until 1972, more than 20 years after Lockhart & Martinelli original paper, that Johannessen (1972) evolved a theoretical model which explains this dependence for stratified flow. Johannessen included some unnecessary simplifications as neglecting the shear stress in the interface and treating only the case where the liquid and the gas are turbulent. Recently Agrawal *et al.* (1973) analysed the pressure drop in horizonal stratified two phase flow in a more adequate fashion but did not arrive at a generalized dimensionless correlation. Russel *et al.* (1974) explored stratified flow utilizing a two dimensional solution for the laminar velocity profile in the liquid. However, they also did not arrive at a generalized dimensionless representation.

In this note we use the basic model suggested by Agrawal *et al.*, in order to examine the conditions under which the dimensionless pressure drop ϕ_G and the hold up R are unique functions of the Lockhart–Martinelli parameter X. It is shown that under most conditions of interest ϕ depends uniquely on X for stratified flow and the exact relationship is developed for laminar or turbulent flow of either phase.

2. ANALYSIS

Consider equilibrium stratified flows as shown in figure 1. A simple momentum balance on each phase yields

$$-A_L\left(\frac{\mathrm{d}P}{\mathrm{d}x}\right) - \tau_{WL}S_L + \tau_i S_i = 0 \qquad [1]$$

$$-A_G\left(\frac{\mathrm{d}P}{\mathrm{d}x}\right) - \tau_{WG}S_G - \tau_i S_i = 0. \qquad [2]$$

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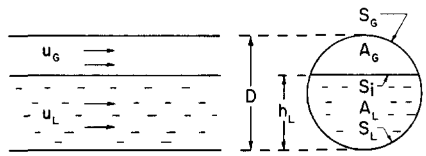

Figure 1. Equilibrium stratified flow.

In these equations A is the flow area of each fluid τ_W is the stress at the wall and S is the perimeter over which the stress acts. τ_i is the interfacial stress acting on S_i which is in the positive x direction for the liquid and in the negative direction for the gas.

Since the pressure gradient $\mathrm{d}P/\mathrm{d}x$ in both phases is assumed to be equal [1] and [2] can be solved for the gradients and equated:

$$\tau_{WG}\frac{S_G}{A_G}-\tau_{WL}\frac{S_L}{A_L}+\tau_i S_i\left(\frac{1}{A_L}+\frac{1}{A_G}\right)=0. \qquad [3]$$

The shear stresses are evaluated in a conventional manner

$$\tau_{WL}=f_L\frac{\rho_L u_L^2}{2};\quad \tau_{WG}=f_G\frac{\rho_G u_G^2}{2};\quad \tau_i=f_i\frac{\rho_G(u_G-u_L)^2}{2} \qquad [4]$$

where ρ and u are the density and average velocity of each component and f is the friction factor. Normally for equilibrium flow $u_G \gg u_L$ such that u_L can be neglected in [4].

Equations [3] and [4] can be conveniently written in a dimensionless form. The reference variables are chosen as: D for length, D^2 for area, the superficial velocities u_{Ls} and u_{Gs} for the liquid and gas velocities, respectively. Designating the dimensionless quantities by a tilde, [3] with shear stresses obtained from [4] takes the form:

$$-\frac{f_L}{f_G}\frac{(\rho_L u_{Ls}^2/2)}{(\rho_G u_{Gs}^2/2)}\frac{\tilde{S}_L}{\tilde{A}_L}\tilde{u}_L^2+\left[\frac{\tilde{S}_G}{\tilde{A}_G}+\frac{f_i}{f_G}\left(\frac{\tilde{S}_i}{\tilde{A}_L}+\frac{\tilde{S}_i}{\tilde{A}_G}\right)\right]\tilde{u}_G^2=0 \qquad [5]$$

Further information is required for the friction factors f_L, f_G and f_i. A widely used method for the correlation of the liquid and gas friction factors is in the form of Blasius equation:

$$f_L=C_L\left(\frac{D_L u_L}{\nu_L}\right)^{-n};\quad f_G=C_G\left(\frac{D_G u_G}{\nu_G}\right)^{-m} \qquad [6]$$

where D_L and D_G are the hydraulic diameters. The liquid is visualized as if it is flowing in an open channel and $D_L=4A_L/S_L$. The gas is visualized as flowing in a closed duct and thus $D_G=4A_G/(S_G+S_i)$. Substituting [6] into [5] results in

$$-X^2\frac{(\tilde{u}_L\tilde{D}_L)^{-n}}{(\tilde{u}_G\tilde{D}_G)^{-m}}\frac{\tilde{S}_L}{\tilde{A}_L}\tilde{u}_L^2+\left[\frac{\tilde{S}_G}{\tilde{A}_G}+\frac{f_i}{f_G}\left(\frac{\tilde{S}_i}{\tilde{A}_L}+\frac{\tilde{S}_i}{\tilde{A}_G}\right)\right]\tilde{u}_G^2=0 \qquad [7]$$

where

$$X^2=\frac{\dfrac{4C_L}{D}\left(\dfrac{u_{Ls}D}{\nu_L}\right)^{-n}\dfrac{\rho_L(u_{Ls})^2}{2}}{\dfrac{4C_G}{D}\left(\dfrac{u_{Gs}D}{\nu_G}\right)^{-m}\dfrac{\rho_G(u_{Gs})^2}{2}}=\frac{(\mathrm{d}P/\mathrm{d}x)_{Ls}}{(\mathrm{d}P/\mathrm{d}x)_{Gs}}. \qquad [8]$$

Equation [7] may be considered as an equation for the equilibrium liquid elevation, $h_L/D=\tilde{h}_L$

and the problem is first to find this quantity. All the dimensionless variables depends on $\tilde{h}_L$ alone as evident from the following relations:

$$\tilde{A}_L = 0.25\{\pi - \cos^{-1}(2\tilde{h}_L - 1) + (2\tilde{h}_L - 1)\surd[1 - (2\tilde{h}_L - 1)^2]\}, \quad [9]$$

$$\tilde{A}_G = 0.25\{\cos^{-1}(2\tilde{h}_L - 1) - (2\tilde{h}_L - 1)\surd[1 - (2\tilde{h}_L - 1)^2]\}, \quad [10]$$

$$\tilde{S}_L = \pi - \cos^{-1}(2\tilde{h}_L - 1), \quad [11]$$

$$\tilde{S}_G = \cos^{-1}(2\tilde{h}_L - 1), \quad [12]$$

$$\tilde{S}_i = \surd[1 - (2\tilde{h}_L - 1)^2], \quad [13]$$

$$\tilde{u}_L = \tilde{A}/\tilde{A}_L, \quad [14]$$

$$\tilde{u}_G = \tilde{A}/\tilde{A}_G. \quad [15]$$

Equation [7] shows that $\tilde{h}_L$ depends on the two parameters X and f_i/f_G. If f_i/f_G can be shown to be a constant or uniquely dependent on X then $\tilde{h}_L$ depends only on X. Assume the interfacial friction factor could be correlated by an equation of the form

$$f_i = C_i\left(\frac{u_G D_G}{\nu_G}\right)^{-m}. \quad [16]$$

Then, comparing [6] and [16] it follows that f_i/f_G indeed is a constant (C_i/C_G) and the solution for $\tilde{h}_L$ by [7] will depend only on the Lockhart-Martinelli parameter X.

Since $\tilde{h}_L$ is a function only of X it is clear that the hold up R which is directly related to $\tilde{h}_L$ is a function of X alone. Similarly the Martinelli dimensionless pressure drop variable ϕ_G is also a function of X alone as evident from the relation

$$\phi_G^2 = \frac{1}{4}\tilde{u}_G^2\frac{(\tilde{u}_G\tilde{D}_G)^{-m}}{\tilde{A}_G}\left[\tilde{S}_G + \frac{f_i}{f_G}\tilde{S}_i\right] \quad [17]$$

which is obtained from [2].

Bergelin & Gazley (1949) reported that indeed for a smooth surface $f_i \simeq f_G$. Others (Hanratty & Engen 1957; Ellis & Gay 1959; Smith & Tait 1966 and Davis 1969) reported different correlations of the friction factor coefficient f_i, especially when the interface is wavy. In those cases the $\tilde{h}_L$ as well as the holdup and ϕ_G cannot depend on X alone. However, the error in the value of $\tilde{h}_L$ incurred by assuming that $f_i/f_G \simeq$ constant is expected to be small. Thus a plot of the holdup and ϕ_G as a function of X should have a universal character as long as the flow is stratified.

3. RESULTS

A solution of [7] using $f_i = f_G$ and the values $C_G = C_L = 0.046$, $n = m = 0.2$ for turbulent flow and $C_G = C_L = 16$, $n = m = 1.0$ for laminar flow is shown in figure 2. In this figure $h_L/D = \tilde{h}_L$ is plotted for the four possible combinations of flow conditions. It is interesting to observe that in spite of the large differences in the behavior between turbulent and laminar flows the differences in the solution for h_L/D are relatively small.

The solution for h_L/D can be easily converted to yield the holdup as a function of X. Figure 3 shows the result as well as the comparison with Johannessen (1972) calculations and some data. First one may observe that the solution for the turbulent-turbulent case and the turbulent liquid–laminar gas are essentially indistinguishable on figure 3. Likewise the cases of laminar–laminar and laminar liquid–turbulent gas are essentially the same. The prediction of Johannessen in which interfacial friction is not accounted for, is slightly higher than this new

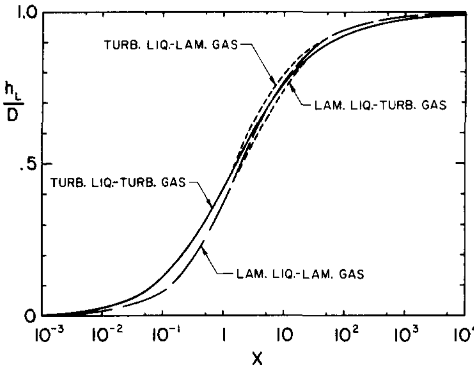

Figure 2. Equilibrium liquid level.

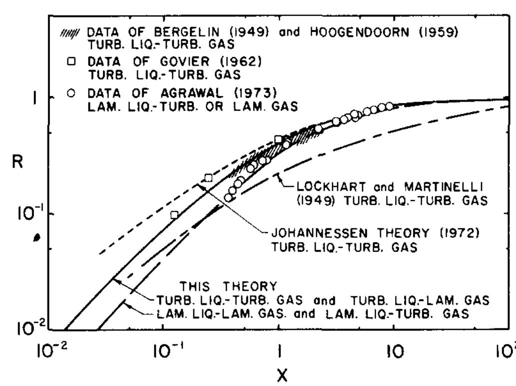

Figure 3. Holdup.

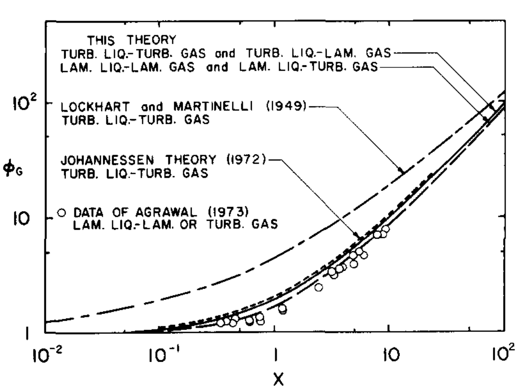

Figure 4. Two phase pressure drop.

model especially for low values of X. Figure 3 shows that this theory is somewhat closer to the experimental data than the solution of Johannessen. The data shown includes experiments on turbulent–turbulent flow by Bergelin & Gazley (1949), Hoogendoorn (1959) and Baker (1954); experiments by Agrawal *et al.* (1973) for the case of laminar flow of liquid with both turbulent as well as laminar flow of air. The correlation by Lockhart–Martinelli is also shown in figure 3 and this predicts holdup values which are considerably lower than experiments.

Figure 4 shows the theoretical relationship between ϕ_G and X. Again it is seen that the prediction for all four flow conditions (with respect to turbulent or laminar flow) are quite close at any single value of X. Data for the turbulent–turbulent case by Bergelin & Gazley (1949), Hoogendoorn (1959) and Baker (1954) are shown by Johannessen to fit his prediction closely. As is seen, his curve for these conditions agrees very nearly exactly with the new results developed here. The individual data points for turbulent-turbulent flow are omitted for clarity. The recent data of Agrawal for the laminar oil and turbulent and laminar air is included to show the excellent fit with the theory presented here. The Lockhart–Martinelli correlation predicts considerably higher values of the dimensionless pressure drop ϕ_G. However, this correlation is assumed to apply under conditions, other than stratified flow where larger pressure drops may be expected.

4. CONCLUSIONS

It has been demonstrated that the hold up and the dimensionless pressure drop for stratified flow are unique functions of X under the assumption that $f_G/f_i \simeq$ const. The further assumption that $f_G = f_i$, namely that the friction factor of the gas at the interface and the gas at the pipe wall are equal is shown to give a result which agrees well with experimental data. While these results are valid only for stratified flow it suggests that the unique dependence between R, ϕ and X has a sound basis in theory.

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